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Worked out Examples to Improve Student Understanding of Area and Perimeter

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Objective/Purpose:

Geometry is a challenging subject to teach and understand. One of the reasons for this is that it involves arithmetic reasoning as well as logical and spatial skills. A key concept taught in Geometry is area and perimeter, and research has found this concept to be challenging for students; often times the teaching of this concept involves an emphasis on the use of formulas leading to misunderstandings of the spatial concept of area and perimeter (Zacharos, 2006 & Hirstein, 1981). This can lead to students using the wrong formula to calculate the required area or perimeter of a given figure. Another common issue is that students often confuse area and perimeter (Kidman, 1999; Hirstein, 2006). In mathematics, area and perimeter are often used as the basis for algebraic problems and to illustrate algebraic concepts such as completing the square. Thus, students need to move beyond a procedural understanding of area and perimeter to a conceptual understanding in order to properly calculate the measurements and apply the concept to other situations (Hirstein, 1981). This shift from procedural to conceptual knowledge aligns with the common core standards for geometry (CCSS-I 6.G.1, 2012).

One possible solution for countering the common misconceptions in regards to area and perimeter is to use worked out examples. Research has shown that worked out examples can improve students understanding in Algebra and Physics (Sweller & Cooper, 1985; Ward & Sweller, 1990). Building off of this in Geometry, modified work examples can reduce the cognitive load and have a positive effect on learning (Tarmizi & Sweller, 1988). Van Gog et al. (2004) suggests that adding how and why to worked out examples can enhance understanding in an efficient and effective way. This is supported by research that showed adding worked out examples to algebra problems along with self-explanations over conventional problems improved student learning (Booth, Lange, Koedinger, & Newton, 2013). These self-explanations allow students to focus not just on the procedural. but the conceptual understanding as well. The purpose of the present study is to determine whether conceptual knowledge can be improved without impeding procedural knowledge using worked-out examples within a unit on area and perimeter.

Perspective/Theoretical Framework:

Cognitive load theory (Sweller, 2011) suggests that student learning can be hindered when they are required to process extraneous information creating a high cognitive load. Worked out examples can be used to reduce the cognitive load for students and increase their understanding (Tarmizi & Sweller, 1988; Van Gog et al., 2004). By providing worked out examples students focus is directed to the similarities, differences, and steps being used (Van Gog et al, 2004; Sweller & Cooper, 1985). Drawing students' attention to the procedure used to calculate area and perimeter rather than having them determine the correct steps and do the computation can reduce the cognitive work load therefore potentially improving understanding.

Van Gog et al (2004) has suggested providing partially filled out work examples with why and how can enhance student understanding, and a growing body of research recommends incorporating incorrect examples into the mix for students to explain as well (Siegler, 2002; Groβe and Renkl, 2007; Booth et al, 2013). However, there is evidence that the effectiveness of this technique can vary with students' relevant prior knowledge (Groβe and Renkl , 2007; Kalyuga, Chandler, & Sweller, 2001). The present study's emphasis on the geometric concepts of area and perimeter will allow us to determine how students' prior spatial knowledge may impact their learning with examples; it is possible that students with higher vs. lower spatial knowledge may benefit differently from the example-based approach. For example, students with low spatial knowledge may struggle, given the inherent spatial nature of many of the examples. Alternatively, these low students may benefit more from having the spatial information laid out for them so that they do not have to envision it themselves.

Methods

A quantitative experimental study was conducted using the Connected Mathematics unit Covering and Surrounding as the means for the intervention. The Connected Mathematics series was created by Michigan State University in collaboration with a National Science Foundation Grant to create a mathematics curriculum for middle school age students based off of research. The Covering and Surrounding unit introduces the concept of area and perimeter, changing one measurement and keeping the other constant, and finding area and perimeter of polygons, irregular shapes, and circles. The participants of the study were 232 sixth grade students in an inner-ring suburban school in the Midwest. Students who missed either the pre-test or post-test were eliminated from the data. There were 210 students used for the analysis out of which 47% were female, 53% were male, 55% were Caucasian, 3% Asian, 57% African-American, 29% Latino, and 3% mixed race. The students were in ten different classes with two different teachers. Classes from each teacher were randomly chosen to be the control group resulting in 97 students in the control and 114 in the experimental group.

Students were given a pretest at the start of the unit on Covering and Surrounding. Teachers used the original Connected Mathematics materials for class lessons. For homework, the control group used the original workbook while students in the experimental group used revised version of the Connected Mathematics workbook. In the revised version, approximately one third of the homework problems were replaced with worked out examples for the student to study and/or explain. Three types of examples were used. The first type showed the correct work and answer to the proposed question with another question at the end asking the student to explain the work that was done. The second type were problems worked out incorrectly and asking students to explain what mistakes were made or to fix the mistake. The third type is filling in the blank with part of the problem done correctly and student is to fill in missing pieces of information. In the book there were 28 correct examples, 5 examples, and 21 partially completed examples. Upon completion of the unit, students were given a posttest identical to the pretest.

Data Source:

Each student in the study took the same version of a paper- and-pencil test before the unit (pre test) and after completing the unit (posttest). The test contained questions on spatial reasoning, conceptual understanding, procedural understanding, and terminology in regards to the Geometry topic of perimeter and area.

The spatial reasoning contained questions on rotation and folding. Students' answers were analyzed as correct, partially correct, or incorrect. The conceptual understanding section contained yes and no questions asking students to compare two figures perimeter and areas. For this section answers were marked as correct or incorrect. The procedural section of the text required students to find area and perimeter of different figures using formulas taught in the unit (formulas were not provided at test time). Students were given partial credit if the wrong answer was a result of an arithmetic error, such as the wrong place for a decimal point. The terminology section asked students to circle the correct term based on a drawing. Students were given points for the correct answers only. Scores were computed for each of the four sections of the pretest and posttest using the percentage of problems answered correctly for each section.

Results

To determine whether condition, prior spatial knowledge, or the interaction between spatial knowledge and condition were predictive of students' learning, we conducted a series of three regression analyses: one for Procedural scores, one for Conceptual scores, and one for Terminology scores. In each case, we entered pretest scores for that particular component, as well as condition, pretest spatial scores, and the interaction between condition and pretest spatial score. As can be seen in Table 1, neither condition nor spatial scores, nor their interaction contributed significant variance toward predicting gain in either procedural or terminology scores. However, each of the factors was predictive of conceptual gain. Being in the example-based condition and having higher prior spatial scores both predicted increases in conceptual scores. To interpret the significant interaction between condition and spatial scores, we conducted separate regressions on posttest conceptual scores for each condition. In the control condition, both pretest conceptual scores ($\beta = .34$, $p < .01$) and spatial scores ($\beta = .27$, $p < .01$) contributed significant variance; having higher conceptual and spatial scores at pretest predicted higher conceptual scores at posttest. For the experimental condition, having higher conceptual scores at pretest predicted higher conceptual scores at posttest ($\beta = .27$, $p < .01$). However, pretest spatial scores were not predictive of posttest conceptual scores ($\beta = .05$, $p = .57$), suggesting that higher pretest spatial scores were not necessary for strong conceptual posttest performance.

Table 1: Regression of posttest Procedural, Conceptual, and Terminology Scores on their Pretest Scores, Condition, Spatial Scores, and the Interaction Between Condition and Spatial Score.

Factor:	Procedural		Conceptual		Terminology	
	β	Significance	β	Significance	β	Significanc
Pretest Score[1]	.49	$p < .01$	.31	$p < .01$	.37	$p < .01$
Condition	.08	$p = .54$	.40	$p < .01$	-.07	$p = .63$
Spatial Score	.31	$p = .11$	.63	$p < .01$	-.04	$p = .85$
Condition x Spatial Score	-.28	$p = .22$	-.58	$p = .02$	.19	$p = .45$

Note: [1]The matching pretest score was utilized in each analysis (i.e., for the procedural analysis, pretest procedural scores were used)

Scholarly Significance

Area and perimeter are difficult concepts for students to grasp, and failure to master these concepts can hinder Algebraic understanding. In this study it was shown that students' prior knowledge has a significant effect on students post understanding across terminology, conceptual understanding, and procedural knowledge for area and perimeter. Past research has shown that worked out examples can improve students' with low prior knowledge understanding of the topic (Sweller & Cooper, 1985). In this study the worked examples did not have an impact based on students' prior knowledge, but their spatial reasoning. In particular, students with low pre spatial reasoning appeared to benefit from the worked out examples, since their spatial reasoning was not a predictor of their post test scores for the experimental group. This suggests that for Geometry, worked out examples should be provided for students with low spatial reasoning.

Future research should consider the question, can worked out examples be used to improve conceptual understanding of students with low spatial knowledge in various Geometry topics? This line of inquiry could lead to the possibility of a new tool to assist students in Geometric understanding. By continuing to study the potential of worked out examples educators can determine the most effective way to incorporate this tool into various curriculums and materials.

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