What Could Go Wrong? Error Anticipation Relates to Conceptual and Procedural Knowledge in Algebra Students.

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Purpose

Algebra students' continued deficiencies in the problem-solving domain, coupled with an increased emphasis on these areas in assessment highlight the importance of exploring novel ways to strengthen students' problem-solving ability through evolving instructional and assessment practices. Mathematics problem –solving has been defined as an interaction between conceptual and procedural knowledge (Rittle-Johnson & Alibali, 1999). While the relationship is considered bidirectional, it has been shown that increases a focus on conceptual knowledge and instruction to be of greater benefit to both procedural knowledge and overall problem-solving performance (Rittle-Johnson & Alibali, 1999; Booth, Koedinger, & Siegler, 2007).

A major hindrance to students' ability to acquire the requisite skills for solving multi-step equations are long-held misconceptions they carry into their Algebra I class that are resilient in the face of instruction (Booth, Koedinger, & Siegler, 2007). These misunderstandings can linger with students to their college years (Prather & Alibali, 2008). It is therefore essential that researchers identify new avenues upon which to attack any misconceptions and identify innovative pedagogical moves that teachers can quickly and easily implement to bolster students' Algebraic conceptual and procedural understanding.

An example of such a unique approach is the recent research into the effectiveness of using correct and incorrect worked examples and self-explanation to clear up students' mathematical misconceptions (Durkin & Rittle-Johnson, 2012; Booth & Koedinger, 2009). This study builds on previous research which found focusing specifically on incorrect worked examples are beneficial for fostering conceptual understanding (Booth, 2013). The purpose of the study is to explore how instruction that focuses on Algebra I students' anticipation and analysis of errors in solving multi-step equations from the third-person perspective can bolster their understanding of algebraic concepts and translate into improved equation-solving performance.

Theoretical Perspectives

This study utilizes an intervention to explore the concepts inherent in Algebraic equation solving by using self-explanation to analyze a problem from a novel perspective. By identifying and addressing the shortcomings in problem–solving strategies through error anticipation and analysis, the study aims to help students streamline their strategy repertoire and advance their thinking.

Siegler's (1996; Chen & Siegler, 2000) Overlapping Waves Theory holds that children think about phenomena in multiple and competing ways. Over time, children experience cognitive growth through the acquisition of new strategies that are mapped onto existing schema, strengthened by repeated use, and refined as efficient means of thinking are embraced to the expense of less-efficient means (Chen & Siegler, 2000; Siegler, 2005).

Self-explanation, defined as explain the cause of an event to oneself, is identified by Siegler (2005) is an easily-administered strategy for promoting learning relative to traditional methods. Students who were asked to explain both correct and incorrect answers to math problems showed greater gains than those exposed to traditional instruction.

This study seeks to examine incorrect worked examples using an unconventional approach that asks participants to anticipate errors another student might make in working through a problem in the absence of the actual incorrect work. The belief is that conceptual and procedural knowledge will benefit by using an iteration of an effective strategy that requires students to thinking deeply about not only how to solve a problem, but the potential missteps he or she may encounter. As these mistakes are highlighted, they will be extinguished as potential strategies so as to streamline students' thought process and strengthen their performance and learning..

Methods

Seventy-five Algebra I students from a suburban middle school in the Midwestern United States participated in the study, 41 females (55%) and 34 males (45%). Fifty-nine percent were African American, 21% were white, 15% were American Indian/Alaskan, 4% were Asian, and 1% were classified as other ethnic background. The study employed a quasi-experimental design where students were assigned to the treatment and control groups according to their rostered section of Algebra I. The four sections of Algebra I were split evenly between two teachers, with each teacher assigned a treatment and experimental group. In all, 37 students (49%) were assigned to the experimental group while 38 (51%) students were assigned to the control group.

Students assigned to the treatment group received Algebra I instruction in a unit on solving multi-step equations with an added emphasis on using self-explanation to predict potential errors in solving a two-step Algebraic equation. Nightly homework assignments included at least one such problem, and additional class time was dedicated to covering similar problems and discussing potential errors. Questions were phrased in the form of "what mistake might a seventh-grader make in solving the equation?" as a means of empowering the students. Students assigned to the control group received traditional instruction.

Data Collection

Students in both groups took identical 30-item pre- and post-tests. The instrument contained questions regarding conceptual and procedural Algebra knowledge. The twenty-one

conceptual items asked students to identify like terms and equivalent expressions, while the nine procedural items asked students to solve multi-step equations, simplify expressions using the distributive property, and anticipate student errors in solving multi-step equations.

Items were scored for accuracy. Scores were recorded as percentage correct on conceptual items, procedural items, and overall. Responses to two of the previously-mentioned items asking for anticipated student errors were coded to classify thematic error variables. The major themes identified were: variables, like terms, negative sign, equals sign, order of operations, other reasonable response, and unreasonable response/answer omission.

Results and Conclusions

A two-level ANCOVA by condition conducted on overall post-test scores when controlling for pre-test performance found a significant effect ($F(2,72) = 4.23$, $p < 0.05$, $\eta p2=0.06$). Students in the treatment group scored an average of 58% versus the control group at 48%. A two-level ANCOVA by condition conducted on post-test performance on procedural items also showed a significant effect ($F2, 72=6.62$, $p<0.05$, $\eta p2=0.08$) when controlled for pre-test performance. Students in the treatment group answered 55% of the procedure items correctly versus 41% for the control group. No significant difference was found on the conceptual scores.

There were no differences by condition on error anticipation items at pretest or posttest. However, coded data from responses to the previously-mentioned error-anticipation items were examined to identify relationships between themes and outcomes at the pre-test stage. The study found that students who provided reasonable responses to the items scored higher on the conceptual items ($r=0.23$, $p<0.05$), procedural items ($r=0.33$, $p<0.01$), and the overall pre-test ($r=0.32$, $p<0.01$). The study also found that students who either provided unreasonable responses or skipped the questions scored lower on conceptual items ($r=-0.23$, $p<0.05$), procedural items ($r=-0.325$, $p<0.01$), and the overall pre-test ($r=-0.321$, $p<0.01$).

Theme-specific correlational analysis also found relationships between student responses and pre-test performance. Students with lower conceptual-item scores on the pretest submitted unreasonable responses or skipped the questions ($r=-0.23$, $p<0.05$). Students whose responses related to variables ($r=0.25$, $p<0.05$) and like terms ($r=0.25$, $p<0.05$) scored higher on pre-test procedural items while those with more unreasonable responses ($r=-0.33$, $p<0.01$) scored lower. On the overall pre-test, students whose responses related to variables ($r=0.26$, $p<0.05$) and like terms ($r=0.26$, $p<0.05$) scored higher while students whose responses involved order of operations ($r=-0.235$, $p<0.05$) or were unreasonable ($r=-0.32$, $p<0.01$) scored lower.

The results indicate students' equation-solving ability benefits from the inclusion of higher-order questions requiring them to contemplate and describe the common errors novice Algebra learners might make. The types of answers offered by stronger students indicate their concern with skills and concepts like variables and like terms that immediately precede equation-solving, while lower-performing students either focus on more-basic Algebra skills like the order of operations or avoid the question altogether. While it makes sense that students who skip questions score lower, the results show that students willing to offer any answers that make sense in the context of the problem is related to better performance.

Significance of the Study

In today's hectic educational environment it is important to identify effective pedagogical moves that ensure a stronger understanding of important concepts and procedures. This study confirms a quick and efficient means of bolstering student success in performing important Algebraic tasks that form the foundation for higher-level maths encountered in high school and beyond—the use of correct and incorrect worked examples with self-explanation.

Future research should consider how to better weave conceptual understanding into error anticipation self-explanation problems. As the coded item from this study was grounded more in procedure, a subsequent study could explore avenues for encouraging deep student thinking regarding conceptual understanding. Future studies should also ascertain what factors lead students to avoid answering error-anticipation questions, which could potentially identify both vital core concepts to be reinforced throughout the Algebra I course as well as avenues of inquiry as to student confidence in working through higher-order self-explanation problems. Such research will help practitioners sort through the noise created by the abundance of information available in our fast-paced world to make sound pedagogical choices in shoring up important Algebraic concepts.

References

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