Integrating Cognitive Principles to Redesign a Middle School Math Curriculum

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Abstract

Does a middle school mathematics curriculum that is redesigned using principles based in cognitive research improve student outcomes? To test whether research can be effectively translated into practice, the Connected Mathematics Project 2 (CMP2) curriculum was revised according to four principles 1) mapping verbal-visual information, 2) interleaving worked examples, 3) spacing learning over time and 4) using formative assessment. This study of 6th grade and 8th grade mathematics education addresses the research question, "Do students who are exposed to specific redesigned CMP2 curriculum modules (treatment) exhibit greater improvements in mathematics performance in the module-specific content area than their counterparts exposed to the regular CMP2 curriculum (control)?" Preliminary analyses show statistically significant effects of the redesigned CMP2 units in three of the four curricular units in this study.

Keywords: cognitive psychology; mathematics; math education; education; spaced learning; formative assessment; worked examples; visual representations

Introduction

Research in cognitive and learning sciences has led to a number of recommendations for improving learning and instruction (Pashler et al., 2007). Although research and theory have led to promising strategies, this substantial knowledge base has had only a limited influence in shaping the design of most educational materials and instructional practices. Further, the majority of studies tend to focus on specific strategies in isolation rather than how principles may be combined to create more effective learning environments. In the current paper, we describe a large-scale effort of the National Center on Cognition and Mathematics Instruction (Math Center) in the United States to bridge research and practice by applying cognitive principles to redesign an existing mathematics curriculum and testing the efficacy of these materials.

To test whether basic research in cognition can meaningfully inform classroom instruction, the Math Center applied four principles to redesign a widely-used middle school (grades 6-8) mathematics curriculum. The Math Center team selected cognitive-based principles shown to improve student learning: 1) integrating visual with verbal information to promote the integration of concepts, 2) structuring practice by interleaving worked examples and self-explanation prompts with new problems to solve, 3) carefully spacing the learning of critical content and skills over time, and 4) providing focused feedback on quizzes and homework.

The *Connected Mathematics Project 2* (CMP2) curriculum is an NSF-funded, research-based curriculum that covers topics emphasized in both national and state standards and aligns well with key ideas from the NCTM (2006) *Focal Points.* Key features of the curriculum are that it (1) is organized around important mathematics ideas and processes, e.g., number sense, symbolic reasoning and probability, (2) is problem-centered, and (3) builds and connects concepts across problems, units, and grades.

Applying the principles to revise instructional materials (e.g., the print curriculum) and instructional practice (e.g., what happens in the classroom) required expertise across many fields. Teams devoted to cognition research, mathematics, professional development, and production collaborated to ensure that the revised materials were grounded in the research findings, were mathematically accurate and appropriate (in terms of student development and curriculum standards), were clearly specified for teachers, and were produced with a high level of technical quality. The iterative, multi-layered design process that we have developed for integrating the cognitive principles with the CMP2 curriculum provides a method for putting research into practice. Although the current work is in the context of mathematics instruction, the approach generalizes to bridging research with instructional design across content areas.

The Principles

The following four principles were selected as they have demonstrated effectiveness in student learning, have broad applicability to instruction, and can be readily implemented in a range of curricular materials.

Integrating Visual and Verbal Information Combining visual information with verbal descriptions serves two important functions in mathematics instruction: 1) ensuring that text for instruction and problem-solving are perceived and understood and 2) promoting fluency in mapping between representations (e.g., equations, diagrams, graphs, or tables). To maximize learning benefits, research suggests that visual and verbal information should be integrated (e.g., Clark & Mayer, 2003; Larkin & Simon, 1987; Moreno & Mayer, 1999) and task irrelevant information should be removed (e.g., Harp & Mayer 1998). Visual cues such as color, proximity and grouping can support integration. Removing "seductive details;" that is, representations that are engaging but only tangentially related to the topic of instruction or the problem at hand (e.g., Harp & Mayer 1998), helps learners focus on relevant information.

Worked Examples In mathematics, students must learn to fluently carry out procedures across a variety of problem types. Interleaving problems to solve with worked examples of how to solve a problem improves student learning (Zhu & Simon, 1987; Clark & Mayer, 2003). Prompting students to explain worked examples, further increases problems by facilitating the integration of new information. (Chi, 2000; Roy & Chi, 2005). In worked example exercises, students see complete or partially worked out solutions (that can be correct or incorrect) and explain the rationale behind problem solving steps or the error that was made in an incorrect example. Positive effects of interleaving worked examples have been reported in a variety of courses (Clark & Mayer, 2003; Paas & Van Merrienboer, 1994; Sweller & Cooper, 1985). Worked examples are more effective and more efficient for learning and transfer because they allow students to spend limited cognitive resources on understanding the ideas underlying the solutions rather than on generating solutions (Sweller, 1999). Further, explaining both correct and incorrect worked examples promotes greater learning than correct examples alone (Siegler, 2002; Siegler & Chen, 2008; Rittle-Johnson 2006).

Spaced Learning Extensive research in cognitive psychology has demonstrated large retention advantages when learners are have multiple opportunities over time to practice key facts, concepts, and knowledge rather than few instances of "massed" practice, a phenomenon called the spacing effect (Cepeda, et al., 2006; Rohrer & Taylor, 2007). When learners practice recalling and applying relevant information, they are more likely to retain that knowledge for a greater period of time. In classroom learning, spacing instruction and practice reinforces connections between key ideas and promotes transfer. In a review of the literature on the spacing effect, Rohrer (2009) argues that practice is neglected in mathematics education.

Formative Assessment Periodic testing provides students with opportunities to practice retrieving knowledge, reflect on the state of their knowledge and transfer knowledge to new problems (Roediger & Karpicke, 2006, Butler & Roediger, 2007). Cycles of feedback and reflection that allow for revision and knowledge updating can help learners master targeted concepts and skills (e.g., Pavlik et al., 2007). Evidence from classroom learning contexts shows that the *formative* use of assessment can enhance instructional effectiveness (e.g., Black & Wiliam, 1998); here, formative assessment is defined as a process used by teachers and students that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes.

Method

The primary research question of this study is: "Do 6th and 8th grade students who are exposed to a redesigned curricular unit (treatment) show greater pre-to-post test improvements in mathematics scores than students exposed to the unmodified curricular unit (control)?"

Participants Fifty-seven 6th grade teachers (1348 students) and 51 8th grade teachers (1265 students) participated in our study. Teachers had prior experience with the CMP2 curriculum and represented a wide diversity of schools across seventeen states in the United States of America. Professional background characteristics of participating teachers and demographic characteristics of their students are presented respectively in Table 1 and Table 2.

Table 1: Professional background of participating teachers.

Characteristic	6th Grade	8th Grade
Majored in math or math education	28%	43%
Advanced degree	63%	67%
Mean years of teaching experience	12.6 (SD = 8.2)	13.7 (SD = 7.8)

Table 2: Demographic characteristics of participating students.

Characteristic	6th Grade	8th Grade
Free/reduced lunch	40%	45%
English language learners	6%	4%
Special education	9%	12%
Ethnicity		
Asian/Pacific Islander	6%	4%
Black	10%	13%
Hispanic	14%	16%
Native American	1%	3%
Multi-ethnic	1%	2%
White	66%	61%
Other	3%	3%

Materials Two 6th grade units and 8th grade units from the CMP2 curriculum were revised according to the cognitive principles described above. The 6th grade units used in this study were Bits and Pieces III (decimals and percents) and Covering and Surrounding (area and perimeter). The 8th grade units were Shapes of Algebra (linear equations and coordinate geometry) and Say it with Symbols (expressions and equations). Teams of researchers were formed for each of the four principles. Initially, the cognitive research teams developed rubrics to identify whether the existing materials aligned with the cognitive design principles, and if not, to specify how the materials would be altered to be in compliance. Next each team made sequential revisions to the CMP2 materials. Changes driven by one set of design principles that overlap with other principles were discussed and resolved in biweekly meetings. Next, the mathematics team reviewed the revised curricular materials to ensure mathematical accuracy and appropriateness. Finally, the production team worked with the cognitive and math content teams to clarify design decisions as necessary. Examples of the original and revised curriculum materials are shown in Figures 1 and 2.

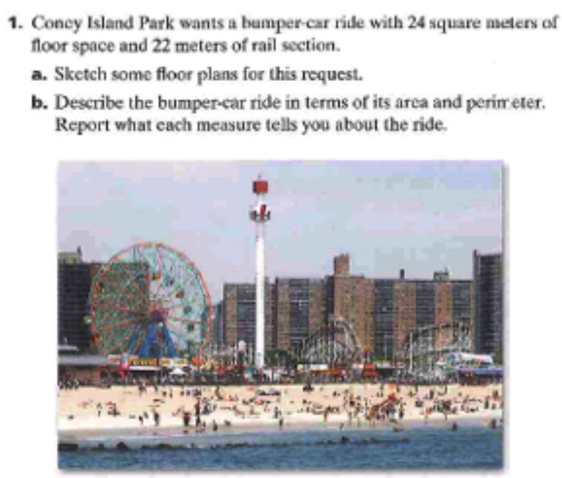

Figure 1: A problem from the original Covering and Surrounding unit.

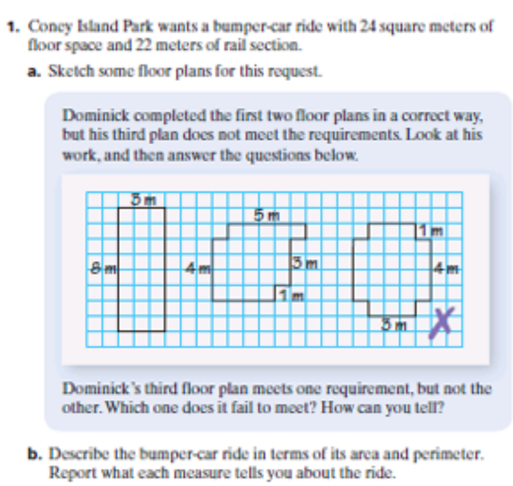

Figure 2: The revised version of the problem in Figure 1. A worked example has been incorporated into part a and the park photograph has been removed.

Concurrent with the production of the materials, the professional development team met to develop measures of fidelity of implementation and to identify effective ways to communicate the underlying rationale and practical implementation of the cognitive design principles to the classroom teachers who will use the redesigned curriculum.

Design We conducted a cluster-randomized control trial to address our research question. We used a within-teacher design and each teacher provided data from two units of CMP2, one revised and one control. Whether a given unit was used in its original or redesigned format was counterbalanced across participants. Teachers were randomly assigned to one of two groups, A and B, as depicted in Table 1 below. Group A served as the experimental group for one of the curriculum units and Group B served as the experimental group for the other. When multiple teachers taught at the same grade level in the same school, half the teachers at the school were assigned to group A and half to group B.

Table 1: Assignment of teachers to group.

Group	Treatment Unit	Control Unit
	6th Grade	
A	Bits and Pieces III	Covering and Surrounding
B	Covering and Surrounding	Bits and Pieces III
	8th Grade	
A	Say it with Symbols	Shapes of Algebra
B	Shapes of Algebra	Say it with Symbols

Procedure All teachers attended a two-day, online, professional development workshop to introduce them to the research-based principles and implications for instructional materials and practice. During these sessions, teachers worked as groups and in pairs to plan instruction for the treatment units. Teachers administered pre-tests for both study units immediately following the professional development. Teachers then taught CMP in their normal curriculum order, administering post-tests immediately upon completion of each study unit, treatment and control. Throughout the study, teachers completed weekly instructional logs describing their use of the CMP curriculum and the principles in the treatment units.

Measures

Researcher-developed assessments were used to evaluate student learning. The content of each curriculum unit was carefully mapped in order to assess the content areas, skills, and contexts presented to students. The same mapping was performed on the assessments to ensure they were well-aligned to the curriculum unit. All items were field tested to establish reliability. Assessments included approximately 16 multiple-choice items and two open-ended items. For each unit, two test forms were created with linking items. Approximately half of the items were derived from existing

CMP materials, and the remaining items were taken from state, national and international standardized tests. Test forms were randomly assigned by class such that half of the classes took form A for pretest and form B for posttest, and the other half of the classes took form B for pretest and form A for posttest. Open-ended items were scored by trained raters using a standardized holistic rubric.

Data Analysis

The results presented in this paper are preliminary; the final paper will present updated analyses. Item response theory (IRT) analysis was used to equate the test scores across forms (Cook & Daniel, 1991). A partial credit model was used to generate item parameters and scale scores[1] for students. ANCOVA models were used to estimate the treatment effects, controlling for pre-test scale scores and ethnicity. ANCOVAs for each unit were performed on students with complete ethnic information and who completed both the pre-test and the post-test for that unit. The ANCOVA sample for each unit is shown in Table 2.

Table 2: Analysis sample for each unit.

Unit	Control	Treatment
	6th Grade	
Covering and Surrounding	546	523
Bits and Pieces III	571	558
	8th Grade	
Shapes of Algebra	416	470
Say it with Symbols	488	509

Results

6th Grade

Students made meaningful gains from pre-test to post-test on both units. To provide context for the IRT scale scores, traditional descriptive statistics for the overall change in students' performance from pre-test to post-test are shown in Table 3.

Table 3: Mean 6th grade assessment performance

Test section	Pre-test	Post-test
	Covering and Surrounding	
Multiple-choice % correct	41.2% (SD = 16.1%)	61.0% (SD = 21.1%)
Open-ended out of 8 points	1.1 (SD = 1.0)	2.1 (SD = 1.3)
	Bits and Pieces III	
Multiple-choice % correct	47.1% (SD = 20.9%)	65.1% (SD = 23.1%)
Open-ended out of 8 points	1.4 (SD = 1.7)	2.5 (SD = 2.1)

[1] Ability estimates were generated using expected a posteriori scoring.

Post-test scale score results by condition and ethnicity are shown in Figure 3 (Covering and Surrounding) and Figure 4 (Bits and Pieces III).

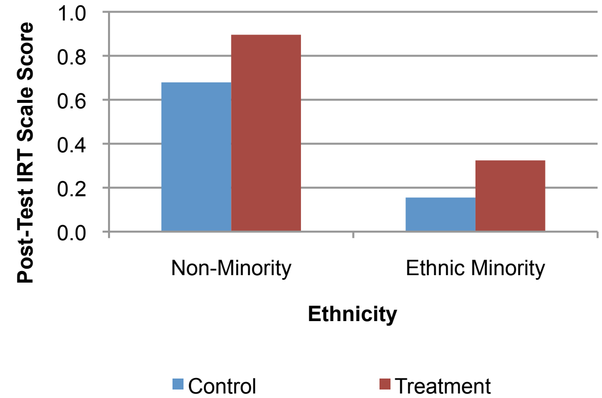

Figure 3: Post-test IRT scale scores for Covering and Surrounding.

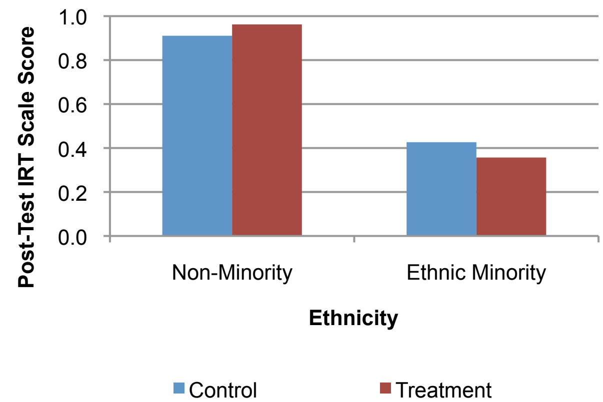

Figure 4: Post-test IRT scale scores for Bits and Pieces III

ANCOVA results are presented in Table 4 (mean-square error is shown in parentheses). Pre-test was significantly associated with post-test scores in both units.

Table 4: 6th grade ANCOVA results

Source	*df*	*F*	*p*	Partial η^2
		Covering and Surrounding		
Pre-test	1	410.43	< .001	0.278
Ethnic minority	1	47.28	< .001	0.043
Treatment	1	13.33	< .001	0.012
Error	1065	(0.48)		
		Bits and Pieces III		
Pre-test	1	490.96	< .001	0.304
Ethnic minority	1	50.86	< .001	0.043
Treatment	1	0.01	.909	< 0.001
Error	1125	(0.57)		

There was also a statistically significant main effect of ethnicity in both units, with non-minority students performing better than members of ethnic minority groups. There was also a statistically significant effect of treatment in Covering and Surrounding, with treatment out-performing control, but no statistically-different differences between groups for Bits and Pieces III.

8th Grade

Again, students made significant gains from pre-test to post-test on both units, although the 8th grade assessments were relatively more difficult than the 6th grade assessments. Traditional descriptive statistics illustrating the overall change in students' performance from pre-test to post-test is shown in Table 5.

Table 5: Mean 8th grade assessment performance

Test section	Pre-test	Post-test
	Shapes of Algebra	
Multiple-choice % correct	37.1% (SD = 15.2%)	51.4% (SD = 20.6%)
Open-ended out of 8 points	0.8 (SD = 1.5)	2.5 (SD = 2.5)
	Say it with Symbols	
Multiple-choice % correct	43.0% (SD = 17.5%)	55.0% (SD = 21.4%)
Open-ended out of 8 points	1.2 (SD = 1.7)	2.4 (SD = 2.5)

Post-test scale score results by condition and ethnicity are shown in Figure 5 (Shapes of Algebra) and Figure 6 (Say it with Symbols). ANCOVA results are presented in Table 6 (mean-square error is shown in parentheses). As in the 6th grade units, pre-test was significantly associated with post-test scores in both units and there was also a statistically significant main effect of ethnicity in both units, with non-minority students performing better than members of ethnic minority groups. Statistically significant effects of treatment were found for both units, with treatment out-performing control. The treatment effect was relatively larger in Shapes of Algebra.

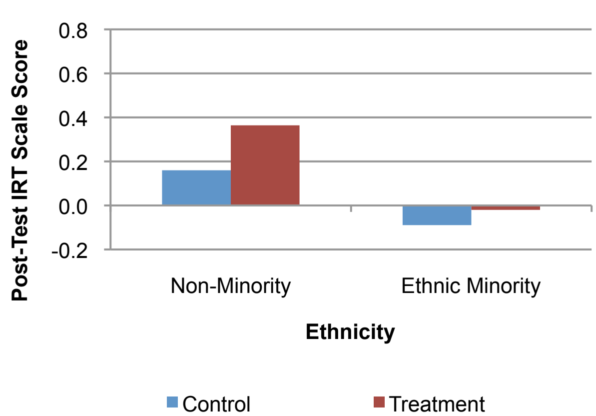

Figure 5: Post-test IRT scale scores for Shapes of Algebra

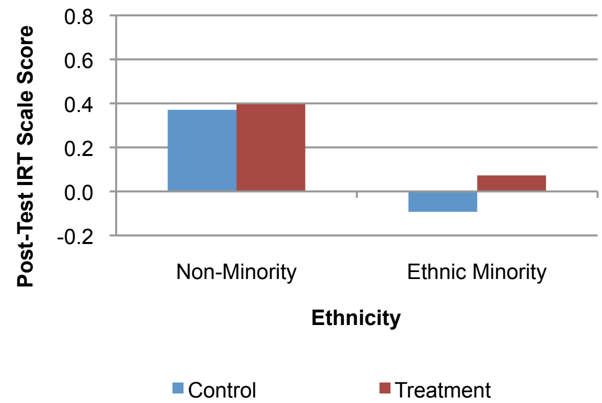

Figure 6: Post-test IRT scale scores for Say it with Symbols

Table 6: 8th grade ANCOVA results

Source	*df*	*F*	*p*	Partial η^2
		Shapes of Algebra		
Pre-test	1	224.80	< .001	0.203
Ethnic minority	1	16.31	< .001	0.018
Treatment	1	9.74	.002	0.011
Error	882	(0.44)		
		Say it with Symbols		
Pre-test	1	474.40		0.323
Ethnic minority	1	11.28	< .001	0.024
Treatment	1	2.44	< .001	0.005
Error	993	(0.46)	.021	

Discussion

Students demonstrated large learning gains for each unit, suggesting both versions of the CMP2 curriculum were effective. Further, three of the four units in this study produced statistically significant effects of the treatment manipulation. That is, the treatment materials produced an additional boost to student learning over and above the existing materials. Why were some treatment units more effective than others? Effects are relatively stronger in Covering and Surrounding and Shapes of Algebra relative to the other grade-level units. One possible explanation for this differential effect is that Covering and Surrounding and Shapes of Algebra are both more spatially-oriented units. Covering and Surrounding addresses area and perimeter and Shapes of Algebra emphasizes coordinate geometry. In contrast, Bits and Pieces III and Say it with Symbols place a greater emphasis on symbolic representations. The more spatially-oriented units may allow for a more potent treatment, as the first cognitive principle directly relates to increasing the coherence in visual representations.

Ongoing analyses will provide further insight into the nature of the treatment effects. We are currently analyzing teachers' instructional logs in order to better understand when and how they implemented the cognitive principles in their teaching practice, aside from using the revised student books. We would expect larger learning gains for students when teachers integrated the principles into classroom practice in addition to giving students the revised books. Additional studies are being carried out at the sites of the partner institutions to investigate effects of the individual principles. During the 2012-2013 and 2013-2014 academic years, the Math Center team is also conducting a cluster-randomized trial of revisions to the entire 7th grade CMP curriculum. If the effects of the principles are cumulative over a school year, we expect greater differences in performance between treatment and control groups.

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